S.6 SEMINAR QUESTIONS AT GAYAZA HIGH SCHOOL P425/1 ON 5TH APRIL 2008 STARTING AT 9:00A.M.

PAPER ONE QUESTIONS.

1. a) Solve for x: $\sqrt{3(x-2)(x-3)} - \sqrt{(x-2)(x-5)} = (x-2)$

b) Find the square root of: $14 + 6\sqrt{5}$

c) Find x if $x^{2.2} = 4.4$

2. a) Prove that if $x + \frac{1}{x} = y + 1$, then $\frac{(x^2 - x + 1)^2}{x(x-1)^2} = \frac{y^2}{y-1}$ hence solve the equation $(x^2 - x + 1)^2 - 4x(x-1)^2 = 0$.

b) Solve the equations: $x^2 + 4xy + y^2 = 13$ and $2x^2 + 3xy = 8$ using $y = mx$.

3. a) If α and β are the roots of $x^2 + px + q = 0$, express $(\alpha - \beta^2)(\beta - \alpha^2)$ in terms of p and q. Hence deduce that for one root to be square the other, then; $p^3 - 3pq + q^2 + q = 0$.

b) Given that the polynomial $f(x) = Q(x)g(x) + R(x)$ where $Q(x)$ is the quotient, $g(x) = (x-\alpha)(x-\beta)$ and $R(x)$ is the remainder, show that $R(x) = \frac{(x-\beta)f(\alpha) + (\alpha - x)f(\beta)}{(\alpha - \beta)}$, when $f(x)$ is divided by $g(x)$.

Hence, find the remainder when $f(x)$ is divided by $x^2 - 9$, given that $f(x)$ divided by $(x-3)$ is 2 and when divided by $(x+3)$ is -3.

4. a) Prove that if: $x = \log_a bc$, $y = \log_b ac$, $z = \log_c ab$ then $x + y + z + 2 = xyz$.

b) Solve for x given that: $\left(\log_5 x\right)\left(\log_5 \frac{2}{x}\right)+1=0$

5. a) Show that the perpendicular distance from a point $P(x_1, y_1)$ to a line $ax+by+c=0$ is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$.

b) Show that the angle between two lines with gradients m_1 and m_2 is given by $\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$.

6. a) PN, the perpendicular from $P(3, 4)$ to the line $2x+3y=1$ is produced to Q such that $NQ=PQ$. Find the coordinates of Q.

b) Find the equations to the lines through the point (2, 3) which makes angles of 45^{o} with the line $x-2y=1$.

7. a) The point (h,k) lies on the curve $y=2x^2+18$. Find the gradient at this point and the equation of the tangent there. Hence, find the equations of the two tangents to the curve which pass through the origin.

b) If: $x^2+2xy+3y^2=1$, Prove that $(x+3y)^3\frac{d^2y}{dx^2}+2=0$

8. a) Differentiate $y=\tan x^2$ form first principles.

b) On a certain curve for which $\frac{dy}{dx}=x+\frac{a}{x^2}$, the point $(2, 1)$ is a point of inflexion. Find the value of a and the equation of the curve.

9. a) A right circular cylinder is inscribed in a sphere of given radius a. Prove that the total area of its surface (including its ends) is $2\pi a^2(\sin 2\theta+\cos^2\theta)$, where $a\cos\theta$ is the radius of an end. Hence prove that the maximum value of the total area is $\pi a^2(\sqrt{5}+1)$.

10. a) Prove that: i) $\dfrac{a+b-c}{a+b+c} = \tan\frac{1}{2}A\tan\frac{1}{2}B$

ii) Prove that $\tan^2\left(\dfrac{\pi}{4}+\theta\right) = \dfrac{1+\sin 2\theta}{1-\sin 2\theta}$, prove also that $\tan 22\frac{1}{2}^o = \sqrt{2}-1$ and $\tan 67\frac{1}{2}^o = \sqrt{2}+1$.

12. a) Solve the equation: $16\sin\theta\cos\theta = \tan\theta + \cot\theta$, for $0^o \le x \le 180^o$

b) Prove that: $8\cos\theta\cos 2\theta\cos 3\theta - 1 = \dfrac{\sin 7\theta}{\sin\theta}$, and find all the angles between 0^{o} and 180^{o} for which $\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}$.

13. a) A contractor finds that his net return from hiring out the machine is decreasing by 10% per annum. If his net gain this year will be $5,000, find the possible total of all the profits.

b) A geometrical sequence has first term 16 and common ratio $3/4$. If the sum of the first n terms exceeds 60, find the possible value of n.

14. a) Prove that $8^n - 7n + 6$ is divisible by 7.

b) Expand $\sqrt{\dfrac{1+5x}{1-5x}}$, as far as and including term in x^3 . Taking the first three terms and $x = \dfrac{1}{9}$, evaluate $\sqrt{14}$ correct to 4 significant figures.

PAPER TWO QUESTIONS.

WORK,POWER AND ENERGY.

15. (a) A bullet of mass 30gm is fired horizontally into a small block of wood of mass 8kg which is suspended by a string 2m long. The bullet remains embedded in the wood and the block rises until the string makes an angle of 30° with the vertical. Find the velocity of the bullet.

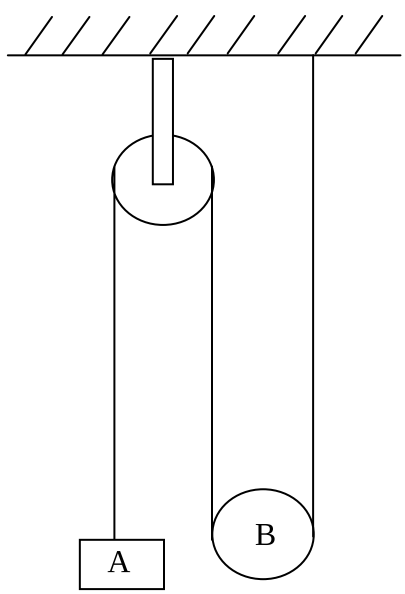
(b) A car of mass 1000 kg moves with its engine shut off down a slope of inclination, θ, where sin θ = 1/20, at a steady speed of 15 ms^{-1}.

(i) Find the resistance to the motion of the car,

(ii) Calculate the power delivered by the engine when the car ascends the same inclination at the same steady speed, assuming that resistance to motion is unchanged.

MOTION.

16.(a) The diagram below shows a particle A of mass 0.5 kg attached to one end of an inextensible light string passing over a fixed light pulley and under a moveable light pulley B, the other end of the string is fixed to a ceiling.



i) What mass should be attached to B for the system to be in equilibrium?

ii) If B is 0.8 kg, what are the accelerations of particle A and pulley B?

(b) A ship A is traveling on a course of 060^0 at a speed of $30\sqrt{3}$ and a ship B is traveling at $20kmh^{-1}$.At noon B is 260km due east of A.

(i) Find the course B must take to come as close as possible to A.

(ii) Find the time when A and B are closest together and the shortest distance.

17. STATICS

(a) ABCD is a square of side 2m. forces of magnitudes 3N, 5N, 7N AND 2N act a long sides DA, AB, BC and CD respectively. Calculate:

(i) the magnitude of the resultant of the forces and the angle made by the resultant with AD.

(ii) the sum of the moments of the forces about A

(iii) the distance from A of the point where the line of action of the resultant of the forces cuts DA produced.

(iv) the equation of the line of action of the resultant.

(b) A uniform ladder 5m long, mass 20 kg rests on a rough horizontal ground and against a smooth vertical wall. It is inclined at an angle of 30^0 to the vertical. Find the normal reactions at each end of the ladder.

18. STATISTICS.

(a) The table below shows the distribution of heights of 60 students in a Maths Class.

Heights (cm)	50 – < 55	55 – < 75	75 – < 85	85 – < 90	90 – < 100	100 – < 120
No. of students	4	20	15	12	7	2

(a) Draw a Histogram and read off the modal.

(b) Construct an Ogive and use it to estimate (i) the median height

(ii) the number of students with height between 60 and 80cm.

(b) A four-man team is to be selected from three women and 4 men,

(a) What is the probability that (i) the women will form the majority

(ii) at least 3 men will be on the committee?

(b) If the group contains a couple that insists on being on the committee together repeat question (a)(i) and (ii) above.

19. NUMERICAL METHODS.

(a)

x	0.1	0.2	0.3	0.4
$\sqrt{x}$	0.3162	0.4472	0.5477	0.6325

Use linear interpolation /Extrapolation to estimate

(i) $\sqrt{0.25}$ (ii) $\sqrt{x} = 0.75$; find the error in each case.

(b) The numbers A and B are rounded off to a and b with errors e_1 and e_2, respectively.

(i) Show that the maximum relative error made in the approximation of A/B by a/b is $e_1/a + e_2/b$.

(i) If also the number C is rounded off to c with error e_3, deduce the expression for the maximum relative error in taking the approximation of

$\frac{A}{B+C}$ as $\frac{a}{b+c}$ in terms of e_1, e_2, e_3 , a , b and c.

(ii) Given that a = 42.326 , b = 27.26 and c = -12.93 are rounded off to the given decimal places, find the range within which the exact value of the expression, $\frac{A}{B+C}$ lies.